SUPERCONDUCTING STATE OF EXCITONIC INSULATOR

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A state of an excitonic insulator with the electric current is studied. Initially, in the metallic phase, the electrons and holes are assumed to be moving in the opposite directions, so as the electric current exists. This state is supported by an external condition (the specimen is in an electric circuit with the current). When the temperature decreases, the transition to the ordering state due to formation of the electron–hole pairs is possible (similar to the ordinary state of the excitonic insulator). The properties of the state at zero temperature are investigated. The spectrum of elementary excitations has a gap, and so the conclusion can be made that obtained state is superconducting one. Thus, depending on the external conditions, excitonic insulator behaves itself like the insulator or superconductor. That is correct in the limit of strong overlapping of the electron–hole pairs.

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It was shown in the work[1] that, in the system of electrons and holes, the Cooper effect[2] is possible due to Coulomb attraction of electrons and holes, and the formation of Bose–condensate of the electron–hole pairs takes place (as in the case of Cooper pairs for superconductors[3]). For the case without the so–called phase fixation, the superfluid motion is possible but, for the system with equal amounts of electrons and holes, there is no electric current for this motion. Therefore it is said on the excitonic insulator rather than superconductor. Nevertheless, it is possible to utilize the superfluidity for the system of spatially separated electrons and holes. This idea was put forward in the work[4].

Interest to such a system is mainly supported because the temperature interval of the ordering state may be greater than for superconductors. It is especially important for the bilayer quantum well systems[4].

Interaction of the electric charges in excitonic insulator is described with the help of dielectric constant (which is large[5]) just as in ordinary insulators, and the same is for a response to alternating electric field[6]. The impression arises that the excitonic insulator behaves entirely as the ordinary dielectric (insulator). However, some difference is possible. In order to make sure in that, one can bring the following argument.

At first, we use a simplified approach and imply that the states of noninteracting electrons and holes (close to extrema of energy zones) may be described by the plane waves with the corresponding effective masses as usual. That is to say, the Hamiltonian H of the system is

$$H = \int d\mathbf{r}\Psi_e^+(\mathbf{r})\left\{\frac{-\triangle}{2m_e} - \mu_e\right\}\Psi_e(\mathbf{r}) + \\ + \int d\mathbf{r}\Psi_h^+(\mathbf{r})\left\{\frac{-\triangle}{2m_h} - \mu_h\right\}\Psi_h(\mathbf{r}) + \\ + H_{e-h} + H_{e-e} + H_{h-h} \ . \qquad (1)$$

Here $\hbar = 1$, $\Psi_e(\mathbf{r})$ is the operator of the electron field, m_e is the effective mass of the electron, μ_e is the chemical potential of electrons which is counted from the appropriate energy minimum (the index h corresponds to the holes). The other terms mark the interactions, for example, H_{e-h} is the interaction of the electrons with the holes.

Let the ground state wave function of an excitonic insulator is $\Phi_0(\mathbf{R}_e, \mathbf{R}_h)$ where the $\mathbf{R}_e$ and $\mathbf{R}_h$ are the sets of the electron and hole coordinates. Let us consider the trial wave function Φ which differs from Φ_0 only by the phase factors, namely

$$\Phi(\mathbf{R}_e, \mathbf{R}_h) = \Phi_0(\mathbf{R}_e, \mathbf{R}_h) \times \\ \times \exp\left\{i\mathbf{p}_e \sum_n \mathbf{r}_n + i\mathbf{p}_h \sum_{n'} \mathbf{r}_{n'}\right\} \qquad (2)$$

(the sums are over the electron and hole coordinates). The phase factors correspond to the independent motions of the electron and hole subsystems.

In order to find the corresponding mean energy E of the system for the state (2), it is enough to determine the one– and two–particle density matrices. For example, the one–particle density matrix for the electrons is

$$\rho(\mathbf{r}', \mathbf{r}) = < \Psi_e^+(\mathbf{r}')\Psi_e(\mathbf{r}) > \ ,$$

$$\Psi_e(\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{p}} a_{\mathbf{p}} \exp(i\mathbf{pr}) \ .$$

Here V is the volume of the system (the area in two–dimensional case), and the symbol $< ... >$ denotes the averaging over the state of the system. One can see that

$$\rho(\mathbf{r}', \mathbf{r}) = \rho_e(\mathbf{r}', \mathbf{r}) \exp\Big[i\mathbf{p}_e(\mathbf{r} - \mathbf{r}')\Big] \qquad (3)$$

(ρ_e corresponds to the ground state of motionless subsystems). The similar phase factors appear in the two–particle density matrix. Therefore, the mean value of potential interactions equals to the value for the motionless subsystems (including the ordering energy). But the kinetic energy changes, and we have

$$\frac{E - E_0}{V} = \frac{p_e^2}{2m_e} \ n_e + \frac{p_h^2}{2m_h} \ n_h \ ,$$

where $n_{e,h}$ are the concentrations of electrons and holes, E_0 is the ground–state energy of motionless subsystems. If the electric charges of electrons and holes are designated by $\pm e$ then the density of electric current can be written as

$$\mathbf{j} = \frac{e\mathbf{p}_e}{m_e}\ n_e - \frac{e\mathbf{p}_h}{m_h}\ n_h\ .$$

For a given value of the current, the total energy is minimal in the case:

$$\mathbf{p}_e = -\mathbf{p}_h \equiv \mathbf{p}_0\ . \qquad (4)$$

The model and results. The arguments cited above show the direction of search only. It is necessary to consider the problem in detail. Let in the metallic phase, when the temperature is higher than the point of the transition, there is a current with electrons and holes moving in opposite directions. Under lowering of temperature, because of Coulomb attraction between electrons and holes, the formation of electron–hole pairs (of Cooper type) is possible just as for motionless subsystems. As a result, the state with a gap in quasiparticle spectrum arise likewise for the ordinary excitonic insulator but with electric current. Apparently, this current is superconducting.

We shall consider the problem by the simplest way. Firstly, we use a model with one type of electrons (corresponding operators of creation and annihilation are $a^+_{\mathbf{p}}$ and $a_{\mathbf{p}}$) and with one type of holes (corresponding operators of creation and annihilation are $b^+_{\mathbf{p}}$ and $b_{\mathbf{p}}$). The spectra and concentrations of electrons and holes are assumed to be the same ($m_e = m_h \equiv m,\ n_e = n_h \equiv n$). Secondly, we introduce an additional condition which takes into account the motions of the electron and hole subsystems. And thirdly, we simplify the model exactly by the same way as in the theory of Bardeen–Cooper–Schrieffer (BCS)[3].

The Hamiltonian of the system is

$$H = \sum_{\mathbf{p}} \xi(\mathbf{p})\Big\{a^+_{\mathbf{p}}a_{\mathbf{p}} + b^+_{\mathbf{p}}b_{\mathbf{p}}\Big\} + \\ +\ \frac{1}{V}\sum W(\mathbf{p}_1 - \mathbf{p}_4)a^+_1 b^+_2 b_3 a_4 + \qquad (5)$$
$$+H_{e-e} + H_{h-h}\ .$$

Here $\xi(\mathbf{p})$ is the energy of particle counted from the Fermi energy, and the interaction operator H_{e-h} is written down explicitly (see (1)). It is necessary to emphasize that, in two–dimensional case, when the electrons and holes are separated by a barrier, its zones may be displaced in energies (this difference may be supported by an external potential between the electron–hole layers).

For the moving subsystems, we add an appropriate condition so as, instead of Hamiltonian (5), we have:

$$H(\mathbf{v}_{e,h}) = \sum_{\mathbf{p}}\Big\{\xi_e(\mathbf{p})a^+_{\mathbf{p}}a_{\mathbf{p}} + \xi_h(\mathbf{p})b^+_{\mathbf{p}}b_{\mathbf{p}}\Big\} + \\ +\ \frac{1}{V}\sum W(\mathbf{p}_1 - \mathbf{p}_4)a^+_1 b^+_2 b_3 a_4 + H_{e-e} + H_{h-h}\ , \qquad (6)$$

$$\xi_{e,h}(\mathbf{p}) = \xi(\mathbf{p}) - \mathbf{p}\mathbf{v}_{e,h} + \frac{m\mathrm{v}^2_{e,h}}{2} = \xi(\mathbf{p} - m\mathbf{v}_{e,h})\ .$$

Here $\mathbf{p}$ is the momentum of particle in the static frame of reference, $\mathbf{v}_{e,h}$ are the velocities of the corresponding subsystems. For zero temperature, the one-particle states with negative energies are filled only.

For arbitrary velocities, the Bose–condensate of electron–hole pairs with nonzero momentum arises. When $m\mathbf{v}_e = -m\mathbf{v}_h \equiv \mathbf{p}_0$ (see (4)), then the pair momentum equals to zero by symmetry. Further we consider just the case, and therefore one can write:

$$\xi_e(\mathbf{p}) = \xi_h(-\mathbf{p}) = \xi(\mathbf{p} - \mathbf{p}_0) \equiv \tilde{\xi}(\mathbf{p})\ . \qquad (7)$$

As a result, we have exactly the same problem as for a superconductor. Therefore one may use the model of BCS type[3], i. e. by taking into account only the part of the interaction which is responsible for the formation of the electron–hole pairs with zero momentum. After that, instead of (6) and bearing in mind of (7), we have:

$$H(\mathbf{v}_{e,h}) \to H(\mathbf{p}_0) = \sum_{\mathbf{p}} \tilde{\xi}(\mathbf{p})\Big\{a^+_{\mathbf{p}}a_{\mathbf{p}} + b^+_{-\mathbf{p}}b_{-\mathbf{p}}\Big\} + \\ +\ \frac{1}{V}\sum_{\mathbf{p},\mathbf{p}'} W(\mathbf{p} - \mathbf{p}')a^+_{\mathbf{p}}b^+_{-\mathbf{p}}b_{-\mathbf{p}'}a_{\mathbf{p}'}\ . \qquad (8)$$

That is our model Hamiltonian.

The well–known self-consistent (mean–field) approximation fits ideally for analysis of the model Hamiltonian (8). In this approximation, the interaction operator H_i (the last term in (8)) is written as

$$H_i \to \sum_{\mathbf{p}}\Big\{a^+_{\mathbf{p}}b^+_{-\mathbf{p}}\Delta(\mathbf{p}) + H.c.\Big\} - \\ -\frac{1}{V}\sum_{\mathbf{p},\mathbf{p}'} W(\mathbf{p} - \mathbf{p}') < a^+_{\mathbf{p}}b^+_{-\mathbf{p}} >< b_{-\mathbf{p}'}a_{\mathbf{p}'} >\ , \qquad (9)$$
$$\Delta(\mathbf{p}) \equiv \frac{1}{V}\sum_{\mathbf{p}'} W(\mathbf{p} - \mathbf{p}') < b_{-\mathbf{p}'}a_{\mathbf{p}'} >\ .$$

Here the symbol $< ... >$ corresponds to averaging over the state of the system.

It is necessary to emphasize that, in this case, the self-consistent approximation gives asymptotically (i. e. in the limit $V \to \infty$) correct result. It is clear from the pseudo–spin approach of Anderson[7] since, in this approach, an every pseudo–spin interacts with all others while its number is macroscopically large so that the fluctuations are not essential. (We remind that the pseudo–spin 1/2 is introduced for the every point of the momentum space and the up and down pseudo–spin projections correspond to the filled and empty states of the electron–hole pair in the point.)

Let us come back to the Hamiltonian (9). The part h of this operator corresponding to the pair of particles with momenta $\pm\mathbf{p}$ has the form:

$$h = \tilde{\xi}(a^+a + b^+b) + \Delta(a^+b^+ + ba)\ . \qquad (10)$$

That is for the real order parameter Δ (the indices are omitted). This operator can be reduced to diagonal form by Bogolyubov transformations, namely:

$$a = u\alpha + v\beta^+ \ , \quad b = u\beta - v\alpha^+ \ ;$$
$$(u^2,\ v^2) = \frac{1}{2}\left\{1 \pm \frac{\tilde{\xi}}{\epsilon}\right\} \ , \quad uv = -\frac{\Delta}{2\epsilon} \ ; \qquad (11)$$
$$\epsilon \to \epsilon(\mathbf{p}) = \sqrt{\tilde{\xi}^2(\mathbf{p}) + \Delta^2(\mathbf{p})} \ .$$

Here α and β are quasi–particle operators of Fermi type and $\epsilon(\mathbf{p})$ is the quasi–particle energy. As a result, the operator h is rewritten in the form:

$$h \to \epsilon(\mathbf{p})(\alpha^+_{\mathbf{p}}\alpha_{\mathbf{p}} + \beta^+_{-\mathbf{p}}\beta_{-\mathbf{p}}) + \left[\tilde{\xi}(\mathbf{p}) - \epsilon(\mathbf{p})\right] \ . \qquad (12)$$

The equation for the order parameter (9) at zero temperature has the form:

$$\Delta(\mathbf{p}) = -\ \frac{1}{V}\sum_{\mathbf{p}'} W(\mathbf{p}-\mathbf{p}')\ \frac{\Delta(\mathbf{p}')}{2\ \epsilon(\mathbf{p}')} \ . \qquad (13)$$

When one recall the definition of $\tilde{\xi}$ (7) then one can see that our order parameter is expressed via the order parameter for motionless subsystems $\Delta_0(\mathbf{p})$ (under the condition $\mathbf{p}_0 = 0$) by the following way:

$$\Delta(\mathbf{p} + \mathbf{p}_0) = \Delta_0(\mathbf{p}) \ . \qquad (14)$$

The same holds for the spectrum of quasi–particles.

The obtained state entirely corresponds to the trial function (2). In order to make sure in that, it is enough to determine, for example, the one–particle density matrix $\rho(\mathbf{r}', \mathbf{r})$ of electrons:

$$\rho(\mathbf{r}', \mathbf{r}) = < \Psi^+_e(\mathbf{r}')\Psi_e(\mathbf{r}) > \ .$$

By passing to the quasi–particle operators (11) and doing the corresponding calculations, we receive the previous result (3) exactly. The same result can be obtained by using the function of BCS type for our case:

$$\Phi = \prod_{\mathbf{p}}\left\{u(\mathbf{p}) + v(\mathbf{p})a^+_{\mathbf{p}}b^+_{-\mathbf{p}}\right\}|0> \ . \qquad (15)$$

The other conclusions are the same as for the trial function (2).

It is necessary to note that the function (15) gives the minimal mean value of the operator (8), i. e. it corresponds to minimum of the energy of the system at the given motions of the subsystems. When our specimen is inserted in an electric circuit with a current then, strictly speaking, only the total current of the system is determined by the external conditions but not the current of every subsystem. However, under given total current, the zero momentum of electron–hole pair corresponds to the minimal energy of the system (see (4)) and just the same was supposed. As long as there is the gap in the excitation spectrum, this current, apparently, flows without resistance.

Discussion. 1) We start with the trial function (2). This function gives a growth of the total energy through the kinetic energies of the subsystems only, and the interaction energy is the same as for the ground state (including the energy of ordering). Formally, such a function can be written for any system but there is no point in doing that in any case. For our problem, it is reasonable only for the case of strong overlapping of electron–hole pairs. In that case, we have the correlated pairs in such a way that, for an electron with some momentum, there is a hole with opposite momentum. That takes place both for the motionless and for the moving subsystems. And so we may use the same model (8) for both cases. It is obvious that the model (8) and the corresponding conclusions are not correct for sufficiently small size of the pair, i. e. if the pair size is smaller than the average distance between the pairs. Indeed, one can imagine a counter–flow of the electrons and holes for continuous medium but can not do it for the individual electron–hole pairs. Apparently, under lowering of concentrations, there is a transition between two regimes, namely, between high concentrations with possibility of nondissipative current (our case) and low concentrations with the insulator properties only.

2) Our state with nondissipative (superconducting) current is supported by an external 'force' – by a current in an electric circuit in which our specimen is included. Therefore, we considered the model with moving subsystems. But when there is no external 'force' then our state become a time-dependent one. Let us imagine that our state is created in an initial point of time and then it is left by itself. We can take our function (15) as initial function and can expand it over the proper states of motionless subsystems. As a result, for the electron current $\mathbf{j}_e(t)$, we find:

$$\mathbf{j}_e(t) = \frac{en\mathbf{p}_0}{m}\ \int_0^\infty \frac{\cos(\sqrt{x^2+1}\ \tau)}{(x^2+1)^{3/2}}\ dx \qquad (16)$$

$$(\tau = 2\Delta_0 t)$$

(and the same result takes place for the holes). This is natural because, in our system without impurities, only total momentum is conserved but not the difference of the momenta of electrons and holes. Apropos, for the state (15), the fluctuations of the current are relatively small ($\sim 1/\sqrt{V}$).

3) A restriction, that spectra of electrons and holes are the same, is not essential. But it is necessary the concentrations of electrons and holes to be sufficiently close. It is known from the theory of superconductivity that, when the difference of concentrations increases, then at first a periodic order state arises and next the ordering disappears at all[8].

4) Another restriction can arise from collisions with impurities. In general, the electrons and holes collide differently with impurities. Therefore, the ordering can vanish for the sufficiently large difference, initially a gapless regime arising[9].

5) It is known that the phase fixation removes the superfluidity in excitonic insulators[10]. But in two–dimensional case, when electrons and holes are separated by sufficiently large barrier, it is insignificant[4]. One can expect that, for our state, the derived results are insensitive to cited effect (in two–dimensional case).

In conclusion, there is good reason to believe that, for an excitonic insulator, an electric current without dissipation is possible. That is shown by using of a simple model. Probably, this state can be observed experimentally in a quasi two–dimensional case when electrons and holes are separated by a barrier (for the bilayer quantum well systems[4], see also the paper[11]).

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